"Integrability in AdS/CFT correspondence"

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Overview

- Introduction to integrability in the AdS/CFT correspondence
  - The $\mathcal{N} = 4$ SYM and asymptotic integrability
  - String theory on $AdS_5 \times S^5$
  - The AdS/CFT correspondence
  - Spectral equations
- Outlook

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The $\mathcal{N}=4$ super Yang-Mills theory

- The $\mathcal{N}=4$ SYM is a four-dimensional gauge theory with four different supersymmetry generators.
- The building blocks of the theory are: Φ_m, $\Psi_{\alpha a}$, $\dot\Psi^a_{\dot\alpha}$ und A_μ.
- The gauge group is $SU(N)$ and the fields transform in the adjoint representation ($U(x) \in SU(N)$)

$$\left\{\Phi;\Psi;\dot\Psi\right\} \mapsto U\left\{\Phi;\Psi;\dot\Psi\right\}U^{-1}, \qquad \mathcal{A}_\mu \mapsto U\mathcal{A}_\mu U^{-1} - ig^{-1}\,\partial_\mu U\,U^{-1}.$$

- The Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\rm YM} \;=\; \mathrm{Tr}\Big(&\frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} + \frac{1}{2}\mathcal{D}^\mu\Phi^n\mathcal{D}_\mu\Phi_n - \frac{1}{4}g^2[\Phi^m,\Phi^n][\Phi_m,\Phi_n] \\ &+\dot\Psi^a_{\dot\alpha}\sigma^{\dot\alpha\beta}_\mu\mathcal{D}^\mu\Psi_{\beta a} - \frac{1}{2}ig\Psi_{\alpha a}\sigma^{ab}_m\varepsilon^{\alpha\beta}[\Phi^m,\Psi_{\beta b}] \\ &-\frac{1}{2}ig\dot\Psi^a_{\dot\alpha}\sigma^m_{ab}\varepsilon^{\dot\alpha\dot\beta}[\Phi_m,\dot\Psi^b_{\dot\beta}]\Big).\end{aligned}$$

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- Due to a large amount of supersymmetries the beta function vanishes and the theory exhibits superconformal symmetry also at the quantum level. The global symmetry algebra gets extended $\mathfrak{so}(1,3) \oplus \mathfrak{so}(6) \to \mathfrak{psu}(2,2|4)$.

- There are no asymptotical distances and thus the physical S-matrix cannot be defined. Correlation functions are well defined. Interesting observables are AD of the composed operators

$$\mathcal{O}(x) = \mathrm{Tr}\,(\Phi\,\Psi\,*\,*\,\ldots)\,,$$

  which receive quantum contributions

$$\Delta(g) = \Delta_0 + \gamma(g)\,.$$

- The full dimensions are eigenvalues of the dilatation operator

$$D\,\mathcal{O}(x) = \Delta_{\mathcal{O}(x)}(g)\,\mathcal{O}(x)\,.$$

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- Huge mixing problem!

$$\mathcal{O}(x) = c_1 \,\mathrm{Tr}\,(\Phi\,\Psi\;*\;*\;\ldots) + c_2\,\mathrm{Tr}\,(\Psi\,\Phi\;*\;*\;\ldots) + \ldots\,.$$

- There exist close subsectors with respect to the action of D. The $\mathfrak{su}(2)$ is the simplest example

$$\mathrm{Tr}\left(\mathcal{X}^M\,\mathcal{Z}^{L-M}\right) + \text{all inequivalent permutations of } \mathcal{X} \text{ and } \mathcal{Z}\,.$$

- The usual perturbative expansion applies

$$D = \sum_n D_{2n}(N) g^{2n}\,.$$

- Even more symmetries appear in the planar limit $(N \to \infty\,, g^2 = \frac{g_{\mathrm{YM}}^2 N}{16\pi^2} = const)$

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- More precisely, the dilatation operator is a member of an infinite family of commuting charges.
  - This was rigorously proven for few first orders of perturbation theory and some subsectors in the asymptotic region ($\ell < L$) .

[N.Beisert '03], [N.Beisert, V.Dippel, M.Staudacher '04],[B.Zwiebel '05], ...

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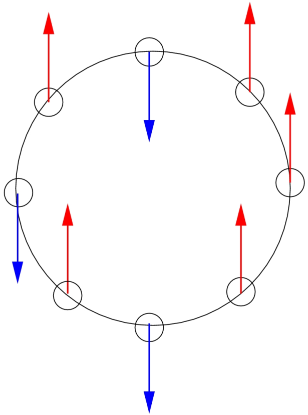
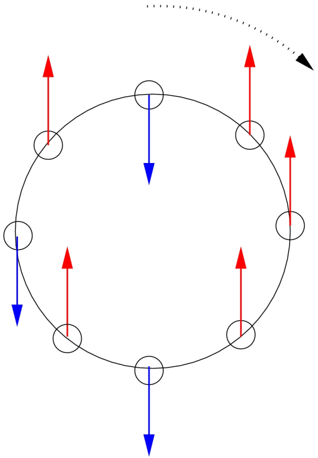
No interactions

$$\mathrm{Tr}\,(\mathcal{X}\,\mathcal{Z}\,\mathcal{Z}\,\mathcal{Z}\,\mathcal{X}\,\mathcal{Z}\,\mathcal{X}\,\mathcal{Z})$$



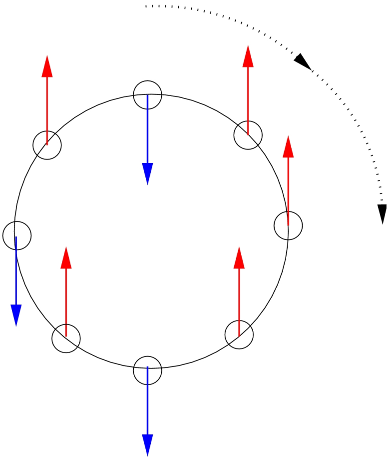
One-loop

$$D_2^{\mathfrak{su}(2)} = \sum_i \frac{1}{2}(1 - \vec{\sigma}_i \vec{\sigma}_{i+1})$$



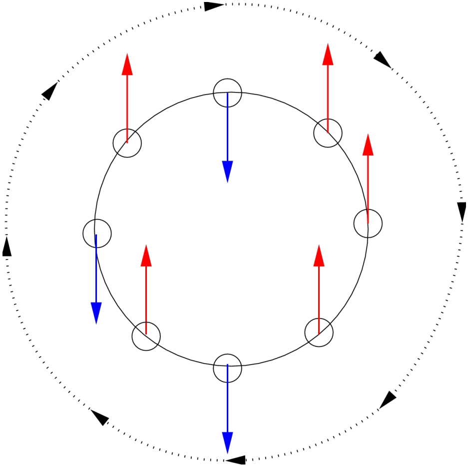
Two-loop

$$D_4^{\mathfrak{su}(2)} = \sum_i (-(1 - \vec{\sigma}_i \vec{\sigma}_{i+1}) + \frac{1}{4}(1 - \vec{\sigma}_i \vec{\sigma}_{i+2}))$$



Wrapping!

$$D_{16}^{\mathfrak{su}(2)} = ???$$



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    - Whether this feature persists for arbitrary operators is still unclear.
- The mixing problem in the asymptotic region can be solved by means of the methods of solid state physics

dilatation operator of the planar $\mathcal{N} = 4$ SYM $=$ Hamiltonian of an integrable spin chain .

- The corresponding spin chain exhibits many novel features, when compared to the usual spin chains considered in the literature: long-rangeness of the interactions increases with the order of the perturbation theory, length fluctuations, ...
- ... but this spin chain is still integrable and can be solved by means of the Bethe ansatz:

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Asymptotic All-Loop Bethe Equations

$$1 = \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}}{u_{1,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - g^2/\ x_{1,k} x^+_{4,j}}{1 - g^2/\ x_{1,k} x^-_{4,j}},$$

$$1 = \prod_{\substack{j=1\\ j\neq k}}^{K_2} \frac{u_{2,k} - u_{2,j} - i}{u_{2,k} - u_{2,j} + i} \prod_{j=1}^{K_3} \frac{u_{2,k} - u_{3,j} + \frac{i}{2}}{u_{2,k} - u_{3,j} - \frac{i}{2}} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}}{u_{2,k} - u_{1,j} - \frac{i}{2}},$$

$$1 = \prod_{j=1}^{K_2} \frac{u_{3,k} - u_{2,j} + \frac{i}{2}}{u_{3,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3,k} - x^+_{4,j}}{x_{3,k} - x^-_{4,j}},$$

$$1 = \left(\frac{x^-_{4,k}}{x^+_{4,k}}\right)^L \prod_{\substack{j=1\\ j\neq k}}^{K_4} \left(\frac{u_{4,k} - u_{4,j} + i}{u_{4,k} - u_{4,j} - i}\, \sigma^2(x_{4,k}, x_{4,j})\right)$$

$$\times \prod_{j=1}^{K_1} \frac{1 - g^2/\ x^-_{4,k} x_{1,j}}{1 - g^2/\ x^+_{4,k} x_{1,j}} \prod_{j=1}^{K_3} \frac{x^-_{4,k} - x_{3,j}}{x^+_{4,k} - x_{3,j}} \prod_{j=1}^{K_5} \frac{x^-_{4,k} - x_{5,j}}{x^+_{4,k} - x_{5,j}} \prod_{j=1}^{K_7} \frac{1 - g^2/\ x^-_{4,k} x_{7,j}}{1 - g^2/\ x^+_{4,k} x_{7,j}},$$

$$1 = \prod_{j=1}^{K_6} \frac{u_{5,k} - u_{6,j} + \frac{i}{2}}{u_{5,k} - u_{6,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{5,k} - x^+_{4,j}}{x_{5,k} - x^-_{4,j}},$$

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- The excitation numbers $K_i, i = 1, \ldots, 7$ are uniquely specified by the eigenvalues of the elements of the Cartan algebra of $\mathfrak{psu}(2,2|4)$.
- The $x^{\pm}(u)$ variables are defined by: [N.Beisert, V.Dippel, M.Staudacher '04]

$$x(u) = \frac{u}{2}\left(1 + \sqrt{1 - \frac{4\,g^2}{u^2}}\right), \qquad x^{\pm} = x(u \pm \frac{i}{2}).$$

- The eigenvalues of the higher conserved charges are given by

$$Q_r = \frac{i}{r-1} \sum_{j=1}^{K_4} \left(\frac{1}{(x^+(u_j))^{r-1}} - \frac{1}{(x^-(u_j))^{r-1}} \right).$$

- The second of these charges Q_2 corresponds to the eigenvalue of the dilatation operator $(D - D_0)$

$$\gamma(g) = 2\,g^2\,Q_2\,.$$

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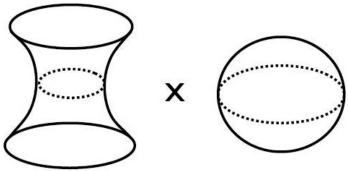
String theory on $AdS_5 \times S^5$ product space



- The IIB string theory on the super coset space $\frac{PSU(2,2|4)}{SO(5)\times SO(4,1)}$ has the same symmetry group as the $\mathcal{N} = 4$ SYM theory.
- The bosonic subspace is $AdS_5 \times S^5$. The radii of the both components are equal (R).
- The integrability of the classical equations of motion has been proven rigorously. [I.Bena, J.Polchinski, R.Roiban '03]
- The quantization of the theory is, however, not understood.
- In some limits semiclassical quantization can be applied.

[S.Frolov, A.Tseytlin '02]

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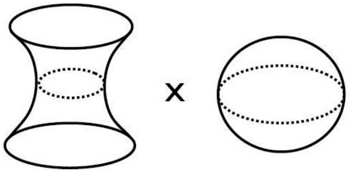

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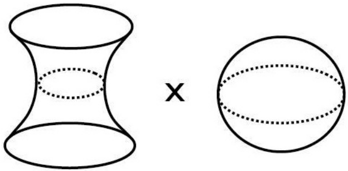

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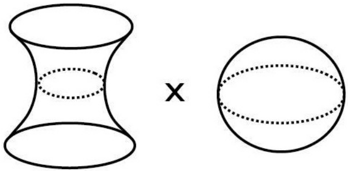

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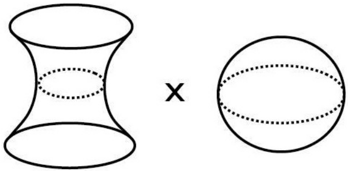

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$$g_s = \frac{4\pi g^2}{N}\,, \qquad \frac{R^2}{\alpha'} = 4\pi g\,. \tag{1}$$

- In particular: $\Delta = E$!
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  - $\Rightarrow$ In the weaker formulation the equivalence is expected in the planar limit ($N \to \infty\,,\ g = const$) only.
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Spectral equations

- Recently, the techniques of Thermodynamic Bethe Ansatz have been applied to the planar AdS/CFT.
- If a two-dimensional sigma model is asymptotically integrable **and** the mirror model ($\tau \leftrightarrow \sigma$) is asymptotically integrable, one can solve the former by determining the spectrum of the mirror model in the infinite volume limit!
- The mirror model for the planar AdS/CFT has been extensively studied and the infinite volume solution (string hypothesis) has been formulated [G. Arutyunov, S. Frolov, 2007], [G. Arutyunov, S. Frolov, 2009]
- The Y-system and TBA equations for the ground state and excited states have proposed by different groups.

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- In its functional form, the spectral equations read:

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- The scaling dimension is then equal to

$$\Delta = \Delta_0 + \sum_j \epsilon_1(u_{4,j}) + \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a^*}{\partial u} \log\left(1 + Y_{a,0}^*(u)\right),$$

with $\epsilon_a(u) = 2ig\left(\frac{1}{x^{[+a]}} - \frac{1}{x^{[-a]}}\right)$ and $f^{[a]}(u) = f(u + ia/2)$.

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- This has already allowed to compute numerous interesting results
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