Zhanna Kuznetsova

Department of Mathematics, Computing and Cognition
Federal University of ABC – São Paulo, Brazil

Supersymmetric extension of the Hopf fibrations and the related non-linear supermultiplets.

Talk given at the Workshop ***: Supersymmetry and Integrable Systems***

Yerevan, August, 24, 2010

- L. Faria Carvalho, Z. K. and F. Toppan, Nucl. Phys B 834 (2010) 237.
- M. Gonzales, Z. K., A. Nersessian, F. Toppan and V. Yeghikyan, Phys.Rev.D80 (2009) 025002.
- Papers on classification of 1D SUSY multiplets.

The four Hopf maps (for $k=1,2,4,8$) can be illustrated by the following commutative diagram

$$\begin{array}{ccc} \mathbf{R}^{2k} & \xrightarrow{p} & \mathbf{R}^{k+1} \\ \rho\downarrow & & \downarrow\rho' \\ \mathbf{S}^{2k-1} & \xrightarrow{h} & \mathbf{S}^{k} \end{array}$$

which connects four spaces (two Euclidean spaces and two spheres) which, for later convenience, can be identified as I, II, III, IV according to

$$\begin{array}{ccc} I & \xrightarrow{p} & II \\ \rho\downarrow & & \downarrow\rho' \\ III & \xrightarrow{h} & IV \end{array}$$

The four arrows correspond to the following maps:

- the bilinear map $p: I \to II$, sending coordinates $\vec{u} \in \mathrm{R}^{2k}$ into coordinates $\vec{x} \in \mathrm{R}^{k+1}$ according to

$$p: \quad \vec{u} \mapsto x_i = u^T \gamma_i u,$$

- the restrictions ρ, ρ' on spheres, where $\rho: I \to III$ and $\rho': II \to IV$;

- the Hopf map $h: II \to IV$, admitting $\mathbf{S}^{k-1}$ as a fiber

The bilinear map $p: \quad \vec{u} \mapsto x_i = u^T \gamma_i u,$

$$\gamma_i \gamma_j + \gamma_j \gamma_i = 2\eta_{ij}$$

This map reflects the connection between the size of gamma matrix (= the number of components in the vector ***u****)*
and the associated to the “metric “ η_{ij} “dimension” D = p + q in (= number of components of the vector ***x****) .*

For $k = 1, 2, 4, 8$ the map preserves the norm, allowing to induce the map h from p:

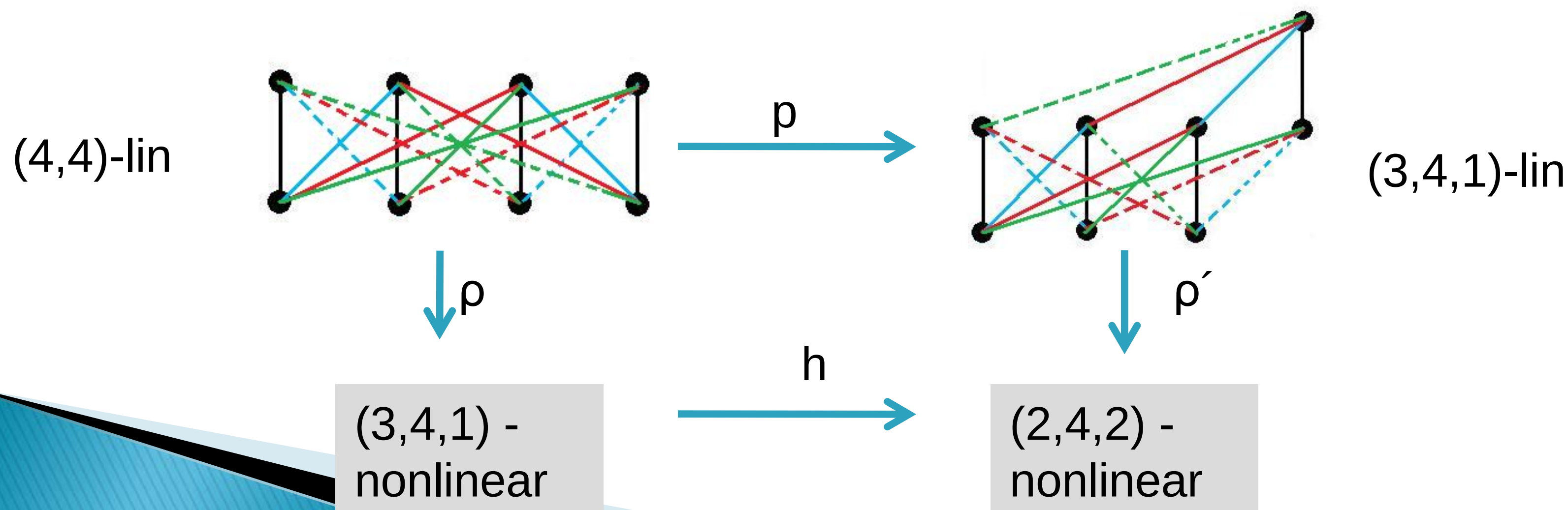
$$u^T u = R \mapsto x^T x = r, \quad with \quad r = R^2.$$

By setting $k = 2^l$, the four Hopf maps h will be referred to (for $l = 0, 1, 2, 3$ respectively) as the $0^{th}, 1^{st}, 2^{nd}$ and 3^{rd} Hopf map.

In the following we will give a detailed description of the supersymmetric extension of the first Hopf map ($k = 2$), corresponding to the diagram

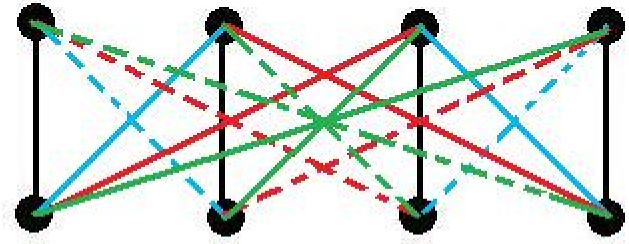
$$\begin{array}{ccc} \mathbf{R}^4 & \xrightarrow{p} & \mathbf{R}^3 \\ \rho \downarrow & & \downarrow \rho' \\ \mathbf{S}^3 & \xrightarrow{h} & \mathbf{S}^2 \end{array}$$

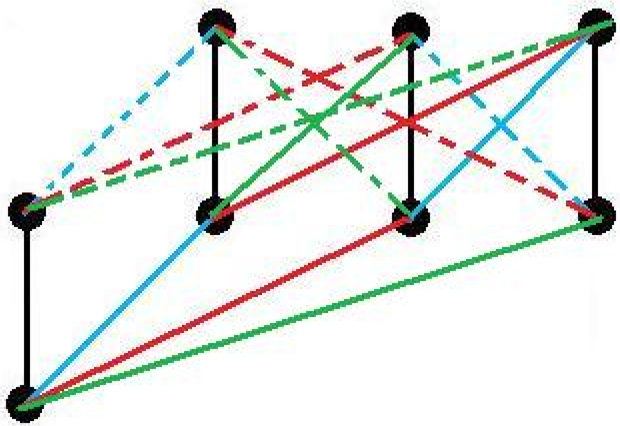
Induced by the $\mathcal{N} = 4$ $(4,4)$ root supermultiplet, three more (inequivalent) $\mathcal{N} = 4$ off-shell supermultiplets are obtained.



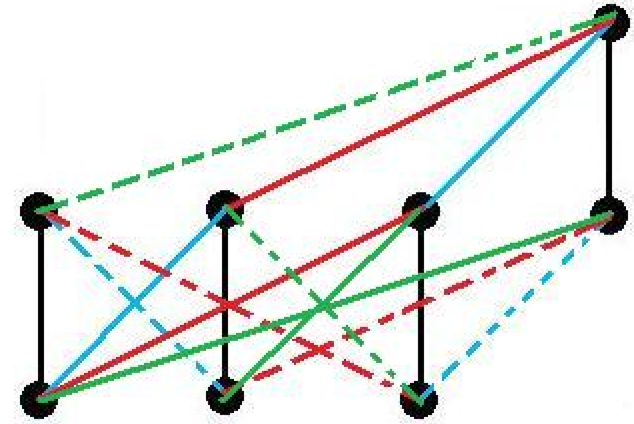
The N=4 irreducible representations:

(4,4) -

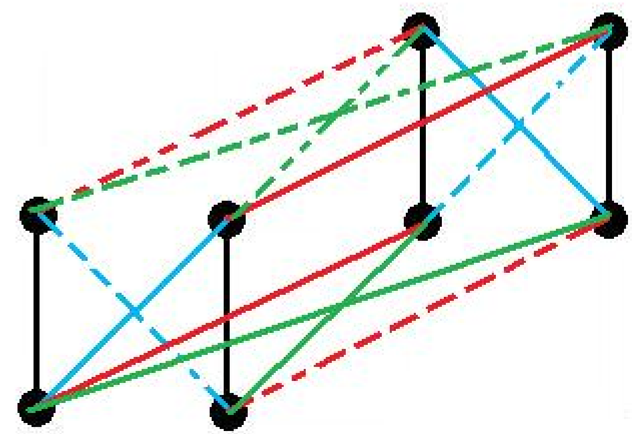




- (1,4,3) ~ (3,4,1) -



(2,4,2) -



$$\begin{array}{ccc} \mathbf{R}^4 & \xrightarrow{p} & \mathbf{R}^3 \\ \rho \downarrow & & \downarrow \rho' \\ \mathbf{S}^3 & \xrightarrow{h} & \mathbf{S}^2 \end{array}$$

$(4,4)_{lin} \rightarrow (3,4,1)_{lin}$: the SUSY extension of the bilinear bosonic transformation.

$$\begin{array}{ccc} (4,4) & \longrightarrow & (3,4,1)_{lin} \\ \downarrow & & \downarrow \\ (3,4,1)_{nl} & \longrightarrow & (2,4,2)_{nl} \end{array}$$

$$\begin{aligned}
x_1 &= 2(u_1u_3 + u_2u_4), \\
x_2 &= 2(u_1u_4 - u_2u_3), \\
x_3 &= {u_1}^2 + {u_2}^2 - {u_3}^2 - {u_4}^2, \\
\mu_1 &= 2(u_1\psi_3 + u_2\psi_4 + u_3\psi_1 + u_4\psi_2), \\
\mu_2 &= 2(u_1\psi_4 - u_2\psi_3 - u_3\psi_2 + u_4\psi_1), \\
\mu_3 &= 2(u_1\psi_1 + u_2\psi_2 - u_3\psi_3 - u_4\psi_4), \\
\mu_4 &= 2(u_1\psi_2 - u_2\psi_1 + u_3\psi_4 - u_4\psi_3), \\
f &= 2(u_1\dot{u}_2 - u_2\dot{u}_1 + u_3\dot{u}_4 - u_4\dot{u}_3) + 4(\psi_1\psi_2 + \psi_3\psi_4).
\end{aligned}$$

The transformation $(4,4) \rightarrow (3,4,1)_{nl}$ is induced after indentifying the three target coordinates entering $(3,4,1)_{nl}$ with the coordinates of the stereographic projection of the S^3 embedded in R^4.

$$\begin{aligned}
w_i &= \frac{Ru_i}{R - u_4}, \quad for \quad i = 1,2,3, \\
\xi_j &= \frac{R\psi_j}{R - u_4}, \quad for \quad j = 1,2,3,4 \\
g &= \frac{R\dot{u}_4}{R - u_4}.
\end{aligned}$$

Similarly, the transformation $(3,4,1)_{lin} \rightarrow (2,4,2)_{nl}$ is induced after identifying the two target coordinates entering $(2,4,2)_{nl}$ with the coordinates of the stereographic projection of the $\mathbf{S}^2$ sphere embedded in $\mathbf{R}^3$. For $(3,4,1)_{lin} \rightarrow (2,4,2)_{nl}$ we have

$$\begin{aligned} z_i &= \frac{rx_i}{r-x_3}, \quad for \quad i=1,2, \\ \eta_j &= \frac{r\mu_j}{r-x_3}, \quad for \quad j=1,2,3,4, \\ h_1 &= \frac{r\dot{x}_3}{r-x_3}, \\ h_2 &= \frac{rf}{r-x_3}. \end{aligned}$$

The last transformation connects the component fields of the two nonlinear supermultiplets. It corresponds to a nonlinear version of the dressing transformation.

$$\begin{aligned} z_i &= \frac{r}{R}w_i, \quad for \quad i=1,2, \\ \eta_j &= \frac{r}{R}\xi_j, \quad for \quad j=1,2,3,4, \\ h_1 &= \frac{r}{R}g, \\ h_2 &= \frac{r}{R}(\dot{w}_3 - w_3 g). \end{aligned}$$

Supersymmetric transformations.

$$\begin{array}{ccc} (4,4) & \longrightarrow & (3,4,1)_{lin} \\ \downarrow & & \downarrow \\ (3,4,1)_{nl} & \longrightarrow & (2,4,2)_{nl} \end{array}$$

Their component fields are parametrized according to

$$\begin{array}{ccc} (u_1,u_2,u_3,u_4;\psi_1,\psi_2,\psi_3,\psi_4) & \longrightarrow & (x_1,x_2,x_3;\mu_1,\mu_2,\mu_3,\mu_4;f) \\ \downarrow & & \downarrow \\ (w_1,w_2,w_3;\xi_1,\xi_2,\xi_3,\xi_4;g) & \longrightarrow & (z_1,z_2;\eta_1,\eta_2,\eta_3,\eta_4;h_1,h_2) \end{array}$$

The greek letters have been employed to denote the fermionic fields; $\vec{u},\vec{x},\vec{w},\vec{z}$ denote the bosonic target coordinates of the respective supermultiplets, while $f,g,h_{1,2}$ denote the auxiliary fields.

$(4,4)_{\text{lin}}$

	Q_1	Q_2	Q_3	Q_4
u_1	ψ_1	ψ_2	ψ_3	ψ_4
u_2	ψ_2	$-\psi_1$	ψ_4	$-\psi_3$
u_3	ψ_3	$-\psi_4$	$-\psi_1$	ψ_2
u_4	ψ_4	ψ_3	$-\psi_2$	$-\psi_1$
ψ_1	$\dot{u}_1$	$-\dot{u}_2$	$-\dot{u}_3$	$-\dot{u}_4$
ψ_2	$\dot{u}_2$	$\dot{u}_1$	$-\dot{u}_4$	$\dot{u}_3$
ψ_3	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_1$	$-\dot{u}_2$
ψ_4	$\dot{u}_4$	$-\dot{u}_3$	$\dot{u}_2$	$\dot{u}_1$

$(3,4,1)_{\text{lin}}$

	Q_1	Q_2	Q_3	Q_4
x_1	μ_1	$-\mu_2$	$-\mu_3$	μ_4
x_2	μ_2	μ_1	$-\mu_4$	$-\mu_3$
x_3	μ_3	μ_4	μ_1	μ_2
μ_1	$\dot{x}_1$	$\dot{x}_2$	$\dot{x}_3$	$-f$
μ_2	$\dot{x}_2$	$-\dot{x}_1$	f	$\dot{x}_3$
μ_3	$\dot{x}_3$	$-f$	$-\dot{x}_1$	$-\dot{x}_2$
μ_4	f	$\dot{x}_3$	$-\dot{x}_2$	$\dot{x}_1$
f	$\dot{\mu}_4$	$-\dot{\mu}_3$	$\dot{\mu}_2$	$-\dot{\mu}_1$

SUSY transformations of $(3,4,1)_{nlin}$ multiplet:

	Q_1	Q_2	Q_3	Q_4
w_1	$\xi_1 + \frac{1}{R}(w_1\xi_4)$	$\xi_2 + \frac{1}{R}(w_1\xi_3)$	$\xi_2 - \frac{1}{R}(w_1\xi_2)$	$\xi_4 - \frac{1}{R}(w_1\xi_1)$
w_2	$\xi_2 + \frac{1}{R}(w_2\xi_4)$	$-\xi_1 + \frac{1}{R}(w_2\xi_3)$	$\xi_4 - \frac{1}{R}(w_2\xi_2)$	$-\xi_3 - \frac{1}{R}(w_2\xi_1)$
w_3	$\xi_3 + \frac{1}{R}(w_3\xi_4)$	$-\xi_4 + \frac{1}{R}(w_3\xi_3)$	$-\xi_1 - \frac{1}{R}(w_3\xi_2)$	$\xi_2 - \frac{1}{R}(w_3\xi_1)$
ξ_1	$\dot{w}_1 - \frac{1}{R}(w_1 g + \xi_1\xi_4)$	$-\dot{w}_2 + \frac{1}{R}(w_2 g - \xi_1\xi_3)$	$-\dot{w}_3 + \frac{1}{R}(w_3 g + \xi_1\xi_2)$	$-g$
ξ_2	$\dot{w}_2 - \frac{1}{R}(w_2 g + \xi_2\xi_4)$	$\dot{w}_1 - \frac{1}{R}(w_1 g + \xi_2\xi_3)$	$-g$	$\dot{w}_3 - \frac{1}{R}(w_3 g + \xi_1\xi_2)$
ξ_3	$\dot{w}_3 - \frac{1}{R}(w_3 g + \xi_3\xi_4)$	g	$\dot{w}_1 - \frac{1}{R}(w_1 g + \xi_2\xi_3)$	$-\dot{w}_2 + \frac{1}{R}(w_2 g - \xi_1\xi_3)$
ξ_4	g	$-\dot{w}_3 + \frac{1}{R}(w_3 g + \xi_3\xi_4)$	$\dot{w}_2 - \frac{1}{R}(w_2 g + \xi_2\xi_4)$	$\dot{w}_1 - \frac{1}{R}(w_1 g + \xi_1\xi_4)$
g	$\dot{\xi}_4$	$-\dot{\xi}_3$	$-\dot{\xi}_2$	$-\dot{\xi}_1$

SUSY transformations of $(2,4,2)_{nlin}$ multiplet:

	Q_1	Q_2	Q_3	Q_4
z_1	$\eta_1 + \frac{1}{r}(z_1\eta_3)$	$-\eta_2 + \frac{1}{r}(z_1\eta_4)$	$-\eta_3 + \frac{1}{r}(z_1\eta_1)$	$\eta_4 + \frac{1}{r}(z_1\eta_2)$
z_2	$\eta_2 + \frac{1}{r}(z_2\eta_3)$	$\eta_1 + \frac{1}{r}(z_2\eta_4)$	$-\eta_4 + \frac{1}{r}(z_2\eta_1)$	$-\eta_3 + \frac{1}{r}(z_2\eta_2)$
η_1	$\dot{z}_1 - \frac{1}{r}(z_1 h_1 + \eta_1\eta_3)$	$\dot{z}_2 - \frac{1}{r}(z_2 h_1 + \eta_1\eta_4)$	h_1	$-h_2 - \frac{1}{r}(\eta_1\eta_2)$
η_2	$\dot{z}_2 - \frac{1}{r}(z_2 h_1 + \eta_2\eta_3)$	$-\dot{z}_1 + \frac{1}{r}(z_1 h_1 - \eta_2\eta_4)$	$h_2 - \frac{1}{r}(\eta_1\eta_2)$	h_1
η_3	h_1	$-h_2 - \frac{1}{r}(\eta_3\eta_4)$	$-\dot{z}_1 + \frac{1}{r}(z_1 h_1 + \eta_1\eta_3)$	$-\dot{z}_2 + \frac{1}{r}(z_2 h_1 + \eta_2\eta_3)$
η_4	$h_2 - \frac{1}{r}(\eta_3\eta_4)$	h_1	$-\dot{z}_2 + \frac{1}{r}(z_2 h_1 + \eta_1\eta_4)$	$\dot{z}_1 - \frac{1}{r}(z_1 h_1 + \eta_2\eta_4)$
h_1	$\dot{\eta}_3$	$\dot{\eta}_4$	$\dot{\eta}_1$	$\dot{\eta}_2$
h_2	$\dot{\eta}_4 - \frac{1}{r}(h_1\eta_4 - h_2\eta_3)$	$-\dot{\eta}_3 + \frac{1}{r}(h_1\eta_3 + h_2\eta_4)$	$\dot{\eta}_2 - \frac{1}{r}(h_1\eta_2 + h_2\eta_1)$	$-\dot{\eta}_1 + \frac{1}{r}(h_1\eta_1 + h_2\eta_2)$

A few comments are in order:

- The non-linearity of the $(3,4,1)_{nl}$ and $(2,4,2)_{nl}$ supertransformations is the mildest possible nonlinearity, since at most bilinear combinations of the component fields appear in the entries.

- The constant parameters R (entering $(3,4,1)_{nl}$) and r (entering $(2,4,2)_{nl}$) can be re-absorbed (set equal to 1) through a suitable rescaling of the component fields. It is however convenient to present them explicitly to show that in the contraction limit (for $R, r \to \infty$) the linear supermultiplets $(3,4,1)_{lin}$ and, respectively, $(2,4,2)_{lin}$ are recovered. As a consequence, $(3,4,1)_{nl}$ and $(2,4,2)_{nl}$ are more general than the corresponding linear supermultiplets with the same field content. Indeed, while the latter can be recovered from the non-linear ones, the converse is not true, as it will be clear from the discussion at the end of the next Section.

Invariant Supersymmetric Actions

The invariant action $S = \frac{1}{m}\int dt\mathcal{L}$ is expressed through the lagrangian $\mathcal{L}$ s.t.

$$\mathcal{L} \;=\; Q_1Q_2Q_3Q_4(F),$$

where F is the unconstrained prepotential.

The invariant Lagrangian for (4,4) multiplet :

$$\begin{aligned}
\mathcal{L}_I \;=\; & \Phi(\dot{u_1}^2 + \dot{u_2}^2 + \dot{u_3}^2 + \dot{u_4}^2 - \psi_1\dot{\psi}_1 - \psi_2\dot{\psi}_2 - \psi_3\dot{\psi}_3 - \psi_4\dot{\psi}_4) + \\
& (\partial_1\Phi)(\dot{u}_2(\psi_1\psi_2 + \psi_3\psi_4) + \dot{u}_3(\psi_1\psi_3 - \psi_2\psi_4) + \dot{u}_4(\psi_1\psi_4 + \psi_2\psi_3)) + \\
& (\partial_2\Phi)(-\dot{u}_1(\psi_1\psi_2 + \psi_3\psi_4) + \dot{u}_3(\psi_1\psi_4 + \psi_2\psi_3) - \dot{u}_4(\psi_1\psi_3 - \psi_2\psi_4)) + \\
& (\partial_3\Phi)(-\dot{u}_1(\psi_1\psi_3 - \psi_2\psi_4) - \dot{u}_2(\psi_1\psi_4 + \psi_2\psi_3) - \dot{u}_4(\psi_1\psi_2 + \psi_3\psi_4)) + \\
& (\partial_4\Phi)(-\dot{u}_1(\psi_1\psi_4 + \psi_2\psi_3) + \dot{u}_2(\psi_1\psi_3 - \psi_2\psi_4) - \dot{u}_3(\psi_1\psi_2 + \psi_3\psi_4)) + \\
& (\Box\Phi)(\psi_1\psi_2\psi_3\psi_4),
\end{aligned}$$

Here the function Φ is determined in terms of the prepotential $F(u_i)$.

$$\begin{aligned}
\Phi \;=\; & \partial_1^2 F\,(u_1,u_2,u_3,u_4) + \partial_2^2 F\,(u_1,u_2,u_3,u_4) + \partial_3^2 F\,(u_1,u_2,u_3,u_4) + \partial_4^2 F\,(u_1,u_2,u_3,u_4) \\
=\; & \Box F\,(u_1,u_2,u_3,u_4)\,. \qquad (24)
\end{aligned}$$

. .

For $(3,4,1)_{\text{lin}}$ the corresponding Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{II} \;=\; & \Phi(\dot{x_1}^2 + \dot{x_2}^2 + \dot{x_3}^2 + f^2 - \mu_1\dot{\mu}_1 - \mu_2\dot{\mu}_2 - \mu_3\dot{\mu}_3 - \mu_4\dot{\mu}_4) + \\ & (\partial_1\Phi)(\dot{x_2}(\mu_1\mu_2 + \mu_3\mu_4) + \dot{x_3}(\mu_1\mu_3 - \mu_2\mu_4) + f(\mu_1\mu_4 + \mu_2\mu_3)) + \\ & (\partial_2\Phi)(-\dot{x_1}(\mu_1\mu_2 + \mu_3\mu_4) + \dot{x_3}(\mu_1\mu_4 + \mu_2\mu_3) - f(\mu_1\mu_3 - \mu_2\mu_4)) + \\ & (\partial_3\Phi)(-\dot{x_1}(\mu_1\mu_3 - \mu_2\mu_4) - \dot{x_2}(\mu_1\mu_4 + \mu_2\mu_3) + f(\mu_1\mu_2 + \mu_3\mu_4)) + \\ & (\Box\Phi)(\mu_1\mu_2\mu_3\mu_4).\end{aligned}$$

where Φ is defined by

$$\Phi \;=\; \partial_1^2 F(x_1,x_2,x_3) + \partial_2^2 F(x_1,x_2,x_3) + \partial_3^2 F(x_1,x_2,x_3) = \Box F(x_1,x_2,x_3)$$

For $(3,4,1)_{\text{nl}}$ we construct the invariant action in terms of prepotential *F(ρ)* that can be expressed as a function of coordinate *ρ*

$$\rho \;=\; \sqrt{w_1^2 + w_2^2 + w_3^2}.$$

The function A(*ρ*) entering the Lagrangian

$$A(\rho) \;=\; \frac{d}{d\rho}F(\rho) \equiv F'$$

$$
\begin{aligned}
\mathcal{L}_{III} \;=\; & \Big[\frac{2}{\rho}A + A'\Big]\big(\dot{w}_1^2 + \dot{w}_2^2 + \dot{w}_3^2 + g^2 - \xi_1\dot{\xi}_1 - \xi_2\dot{\xi}_2 - \xi_3\dot{\xi}_3 - \xi_4\dot{\xi}_4\big) + \\
& \frac{1}{\rho}\Big[\frac{2}{\rho^2}A - \frac{2}{\rho}A' - A''\Big]\Big[\dot{w}_1\Big(w_2(\xi_1\xi_2 + \xi_3\xi_4) + w_3(\xi_1\xi_3 - \xi_2\xi_4)\Big) - \\
& -\dot{w}_2\Big(w_1(\xi_1\xi_2 + \xi_3\xi_4) - w_3(\xi_1\xi_4 + \xi_2\xi_3)\Big) - \dot{w}_3\Big(w_1(\xi_1\xi_3 - \xi_2\xi_4) + w_2(\xi_1\xi_4 + \xi_2\xi_3)\Big) - \\
& -g\Big(w_1\Big(\xi_1\xi_4 + \xi_3\xi_4\Big) - w_2\Big(\xi_1\xi_3 - \xi_2\xi_4\Big) + w_3(\xi_1\xi_2 + \xi_3\xi_4)\Big)\Big] + \Big[\frac{4}{\rho}A'' + A'''\Big]\xi_1\xi_2\xi_3\xi_4 + \\
& \frac{1}{R}\Big\{ - \Big[\frac{2}{\rho}A + A'\Big](w_1\dot{w}_1 + w_2\dot{w}_2 + w_3\dot{w}_3)g - \Big[\frac{4}{\rho}A + 4A' + \rho A''\Big]\cdot \\
& \cdot\Big(\dot{w}_1(\xi_1\xi_4 + \xi_2\xi_3) - \dot{w}_2(\xi_1\xi_3 - \xi_2\xi_4) + \dot{w}_3(\xi_1\xi_2 + \xi_3\xi_4)\Big)\Big\} + \\
& \frac{1}{R^2}\Big\{\rho\Big[2A + \rho A'\Big](\dot{w}_1^2 + \dot{w}_2^2 + \dot{w}_3^2 + 2g^2 - \xi_1\dot{\xi}_1 - \xi_2\dot{\xi}_2 - \xi_3\dot{\xi}_3 - \xi_4\dot{\xi}_4) - \Big[\frac{4}{\rho}A + 4A' + \rho A''\Big]\cdot \\
& \cdot\Big[\dot{w}_1\Big(w_2(\xi_1\xi_2 + \xi_3\xi_4) + w_3(\xi_1\xi_3 - \xi_2\xi_4)\Big) - \dot{w}_2\Big(w_1(\xi_1\xi_2 + \xi_3\xi_4) - \\
& -w_3(\xi_1\xi_4 + \xi_2\xi_3)\Big) - \dot{w}_3\Big(w_1(\xi_1\xi_3 - \xi_2\xi_4) + w_2(\xi_1\xi_4 + \xi_2\xi_3)\Big) + \\
& -2g^2\Big(w_1(\xi_1\xi_4 + \xi_2\xi_3) - w_2(\xi_1\xi_3 - \xi_2\xi_4) + w_3(\xi_1\xi_2 + \xi_3\xi_4)\Big)\Big] + \\
& 2\Big[\frac{4}{\rho}A + 14A' + 16\rho A'' + \rho^2 A'''\Big]\xi_1\xi_2\xi_3\xi_4\Big\} + \\
& \frac{1}{R^3}\Big\{2\rho\Big[2A - \rho A'\Big](w_1\dot{w}_1 + w_2\dot{w}_2 + w_3\dot{w}_3)g - \rho\Big[6A - 6\rho A' - \rho^2 A''\Big]\cdot \\
& \cdot\Big(\dot{w}_1(\xi_1\xi_4 + \xi_2\xi_3) - \dot{w}_2(\xi_1\xi_3 - \xi_2\xi_4) + \dot{w}_3(\xi_1\xi_2 + \xi_3\xi_4)\Big)\Big\} + \\
& \frac{1}{R^4}\Big\{\rho^3\Big[2A + \rho A'\Big]g^2 + \rho\Big[6A + 6\rho A' + \rho^2 A''\Big]\Big(w_1(\xi_1\xi_4 + \xi_2\xi_3) - w_2(\xi_1\xi_3 - \xi_2\xi_4) + \\
& w_3(\xi_1\xi_2 + \xi_3\xi_4)\Big)g + \rho\Big[24A + 36\rho A' + 12\rho^2 A'' + \rho^3 A'''\Big]\xi_1\xi_2\xi_3\xi_4\Big\}. \qquad (29
\end{aligned}
$$

Prepotentials and their associated sigma-models.

The associated sigma-models are constructed by

- consistently setting equal to zero all the fermionic fields in the supermultiplets;
- solving the algebraic equations of motion for the auxiliary fields;
- re-expressing the resulting lagrangians as $L = g_{ij}\dot{X}^i\dot{X}^j$

In our case the metric g_{ij} is obtained from prepotential F.

For the $(3,4,1)_{nl}$ multiplet with prepotential $F(\rho)$ the metric is diagonalized when expressed in terms of the redefined target coordinates.

$$\begin{aligned} \omega_1 &= \rho\cos(\theta_1)\sin(\theta_2) \\ \omega_2 &= \rho\sin(\theta_1)\sin(\theta_2) \\ \omega_3 &= \rho\cos(\theta_2) \end{aligned}$$

The nonvanishing components of the metric are

$$\begin{aligned} g_{\rho\rho} &= \frac{4(\rho^2+1)}{\rho}\left[\rho F''(\rho) + F'(\rho)\right], \\ g_{\theta_1\theta_1} &= \rho(\rho^2+1)\sin(\theta_2)\left[\rho F''(\rho) + F'(\rho)\right], \\ g_{\theta_2\theta_2} &= \rho(\rho^2+1)\left[\rho F''(\rho) + F'(\rho)\right] \end{aligned}$$

SOME OPEN PROBLEMS AND FUTURE INVESTIGATIONS

- **Supersymmetrie extension on 2nd and 3d Hopf maps**

- **Stereographic projection is not covariant with respect to U(1). Sperical coordinates.**

- **Make the fiber local (gauge field).**

- **Classification of SUSY nonlinear.**

Thank you for the attention.